# MEASURING ECONOMIC GROWTH AND ITS RELATION WITH PRODUCTION POSSIBILITY FRONTIER AND RETURNS TO SCALE

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**Abstract**
Economic growth is frequently explained by an increase in the quantity of production. However, economic growth is an increase in the productive capacity of an economy and it is explained by an outward shift of production possibility frontier. The aim of this study is to measure economic growth as an increase in the productive capacity by using production possibility frontier. Our result shows that there is a condition in order to guarantee positive economic growth.

**Keywords:** Economic Growth; Production Possibility Frontier; Returns to Scale.
**JEL:** O40; O49.

## 1. Introduction

Economic growth is not an increase in the quantity of production. Economic growth is an increase in the productive capacity of an economy. This increase in the productive capacity can be explained by using production possibility frontier. The present paper's purpose is to measure economic growth as an increase in the productive capacity by using production possibility frontier. To our knowledge, Nutter (1957) discussed measuring productive capacity of an economy based on production possibility frontier. Our attempt is to contribute this debate by obtaining simple mathematical conditions. These conditions may be useful in order to make a growth analysis compatible with the theory.

Following section gives a short literature on the shape of the production possibility frontier. Third section explains measuring economic growth based on production possibility frontier and gives two propositions. Finally, conclusions are presented.

## 2. Short Literature on the Shape of the Production Possibility Frontier

Economic growth is frequently explained by an outward shift of **concave** production possibility frontier. However, there is an important literature on the shape of production possibility frontier.

Worswick (1957) offers a simple proof of the proposition that the production possibility frontier is convex from above. Green (1959) criticizes Worswick (1957) and shows that assumption made in Worswick (1957) contradicts certain of the conditions of Worswick (1957). Bator (1957: 50-53) examines the impact of increasing returns to scale on the production possibility frontier. According to Bator (1957), if there are increasing returns to scale for both of the producers , then production possibility frontier to be concave to the origin. However, Bator (1957) shows it is also possible that when both production functions exhibit increasing returns to scale, production possibility frontier to be convex to the origin. Herberg and Kemp (1969) analyze the relation between returns to scale and locus of the production possibilities. Herberg and Kemp (1991) briefly explains the main result of the Herberg and Kemp (1969):

"… it was shown that if joint production is ruled out, if each production function is homothetic and if some additional, more technical requirements are met, then in a neighborhood of zero output for any commodity the production frontier is strictly convex (concave) to the origin if and only if that commodity is subject to locally increasing (decreasing) returns to scale".

Melvin (1971) explains a geometric method in order to derive the production possibility frontier. Melvin (1971) uses the representative isoquants of the two commodities, and illustrates the relative shapes of the production possibility frontier. Minabe (1980) shows that if returns to scale do not differ in different output ranges, the production possibility frontier may not be either concave or convex but may be both concave and convex. Minabe (1980: 1) says "it is known that slightly increasing returns to scale in one industry do not necessarily mean that the production possibility curve has the wrong curvature (or convex to the origin)." However Minabe (1982) corrects that statement in the light of Herberg and Kemp (1969). Mayer (1974) examines the shape of the production possibility locus when economies of scale are internal. Mayer (1974) proves that, under some assumptions, if in both industries local returns to scale are non-increasing, the production possibility locus is locally concave to the origin. Panagariya (1980), assuming that economies and diseconomies are both external to the firm and internal to the industry, investigates results of variable returns to scale for the Stolper-Samuelson and Rybczynski theorems. Panagariya (1980: 501) shows that "the validity of the Rybczynski theorem is neiher necessary nor sufficient for the production possibility frontier to be strictly concave." This result can also be repeated for the Stolper-Samuelson theorem. Panagariya (1981: 221) considers a "two-commodity model with increasing returns to scale in one industry and decreasing returns to scale in the other, and discuss … implications of variable returns to scale". Note that economies and diseconomies are external to the firm and internal to the industry. Panagariya (1981: 222) shows that "the production possibility frontier is strictly concave to the origin near … the increasing returns to scale axis and strictly convex to the origin near … the decreasing returns to scale axis". Tawada and Abe (1984), analyze the shape of the production set of an economy under two primary inputs, two consumption goods, and one pure public intermediate good. Tawada and Abe (1984: 233) show that “the frontier is uniformly concave to the origin if the production functions are separable in primary and public inputs and if the elasticity of output with respect to the public input is constant and the same between industries.” Kemp and Tawada (1986) analyze the characteristics of the world production set assuming variable returns to scale and show that, “the world production set has the same properties as the production set of a single closed economy”. Kemp and Tawada (1986: 260) prove that “if in both countries the second industry is subject to increasing returns to scale then … the world production frontier is strictly convex to the origin; and this is so whatever the returns to scale in first industry” and “if in both countries the second industry is subject co decreasing returns to scale then … the world production frontier is strictly concave to the origin; and this is so whatever the returns to scale in the first industry.” Tawada (1989) assumes an economy which has one factor (labour) and two commodities which are tradedable. The factor, labour ,is not mobile among countries, however it is mobile between domestic production sectors. Tawada (1989) also assumes constant returns to scale conditions. Tawada (1989) imposes an additionally assumption such that external economies exist in the production of the first commodity, thus, “the social production of the 1st industry obeys increasing returns to scale… while the 2nd commodity is referred to as the constant returns to scale commodity” (Tawada, 1989: 22). According to these and some other assumptions, “the frontier is negatively sloped and strictly convex to the origin” (Tawada, 1989: 24). Wong (1996), determines two conditions which are required for Herberg-Kemp curvature. Dalal (2006), using a maximum value function, shows that the conditions which are sufficient to assure global concavity of the production possibility frontier are considerably less stringent than those stated in the literature. Dalal (2006) demonstrates that concavity without homotheticity, or non-increasing returns to scale and quasiconcavity without homogeneity are sufficient the generality of existing results. Finally, Mert (2016a) shows that if returns to scale are constant or increasing or decreasing, production possibility frontier can be convex or concave or linear under certain conditions.

Taking into account the textbook explanation of economic growth, the aim of this work is to measure economic growth using **concave** production possibility frontier.

## 3. Measuring Economic Growth Using Production Possibility Frontier

Economic growth occurs when productive capacity of an economy increases. This is shown as an outward shift of **concave** production possibility frontier. Then, economic growth can be measured if this shift is measured. In order to measure this shift, first, assume that Cobb-Douglas production functions of two commodities ( $x$ and $y$ ) are the following:

$$Q_x = A_x K_x^{\alpha_x} L_x^{\beta_x} \tag{1}$$

$$Q_y = A_y K_y^{\alpha_y} L_y^{\beta_y} \tag{2}$$

$Q$ is output, $A$ is level of technology, $K$ is capital stock and $L$ is labour force. $\alpha$ is elasticity of output with respect to capital, and $\beta$ is elasticity of output with respect to labor. The level of technology is constant, identifying assumption is Hicks-neutral, and $\alpha_x, \alpha_y$ , $\beta_x$, $\beta_y$ have positive values.

The isoquant curve for the commodity $x$ and $y$ are the following:

$$K_x = a_x L_x^{-b_x} \tag{3}$$

$$K_y = a_y L_y^{-b_y} \tag{4}$$

where $a$ and $b$ are parameters ( $a_x > 0$ and $b_x > 0$ ; $a_y > 0$ and $b_y > 0$ ).

Since $\frac{dK_x}{dL_x}\frac{L_x}{K_x} = -b_x$ and $\frac{dK_y}{dL_y}\frac{L_y}{K_y} = -b_y$, (5) and (6) can be written.

$\beta_x / \alpha_x = -b_x$

(5) $\beta_y / \alpha_y = -b_y$ (6)

Mert (2016a) shows that in the light of the definitions above, the equation of the production possibility frontier is the following:

$$Q_x \frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} \left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}} \left(\frac{dK_y}{dL_y}\right)^{\alpha_y} \left(\frac{dK_x}{dL_x}\right)^{-\alpha_x} \frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}} = Q_y \tag{7}$$

(7) can be rewritten as:

$$d\left[ Q_x \frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} \left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}} \left(\frac{dK_y}{dL_y}\right)^{\alpha_y} \left(\frac{dK_x}{dL_x}\right)^{-\alpha_x} \frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}} \right] = dQ_y \tag{8}$$

In order to measure the shift, assume that $dQ_y = 0$. Then (9) can be written:

$$d\left[Q_x \frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]=0 \quad (9)$$

Implementing rules, followings occur:

$$\delta Q_x\left[\frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$

$$\delta\frac{A_y}{A_x}\left[Q_x L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$

$$\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[Q\frac{A_y}{A_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$

$$\delta\left[\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\right]\left[Q_x\frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$

$$\delta\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left[Q_x\frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$

$$\delta\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\left[Q_x\frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\right]=0$$

(10)

Since it is assumed $\delta\left(\frac{a_x b_x}{a_y b_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}=0$ , $\delta\left[\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\right]=0$ and $\delta\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}=0$

(see, Mert (2016a)) followings are written:

$$
\begin{aligned}
&\delta Q_x\left[\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&\delta\frac{A_y}{A_x}\left[Q_xL_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[Q\frac{A_y}{A_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&0\left[Q_x\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&0\left[Q_x\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&0\left[Q_x\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\right]=0
\end{aligned}
\tag{11}
$$

$$
\begin{aligned}
&\delta Q_x\left[\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&\delta\frac{A_y}{A_x}\left[Q_xL_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+\\
&\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[Q_x\frac{A_y}{A_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]=0
\end{aligned}
\tag{12}
$$

Rearranging (12):

$$\frac{\delta Q_x}{Q_x}\left[\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$
$$\delta\frac{A_y}{A_x}\left[L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]+$$
$$\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[\frac{A_y}{A_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]=0 \tag{13}$$

$$\frac{\delta Q_x}{Q_x}\left[\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]=$$
$$-\delta\frac{A_y}{A_x}\left[L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right]$$
$$-\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[\frac{A_y}{A_x}\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}\right] \tag{14}$$

Assume that $R=\left(\frac{a_xb_x}{a_yb_y}\right)^{\frac{\beta_y+\alpha_y}{-b_y-1}}\left(\frac{dK_y}{dL_y}\right)^{\alpha_y}\left(\frac{dK_x}{dL_x}\right)^{-\alpha_x}\frac{(-b_x)^{\alpha_x}}{(-b_y)^{\alpha_y}}$:

$$\frac{\delta Q_x}{Q_x}\left[\frac{A_y}{A_x}L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}R\right]=-\delta\frac{A_y}{A_x}\left[L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}R\right]-\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}\left[\frac{A_y}{A_x}R\right] \tag{15}$$

Leaving alone $\frac{\delta Q_x}{Q_x}$:

$$\frac{\delta Q_x}{Q_x} = -\delta \frac{A_y}{A_x} \frac{\left[ L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} R \right]}{\left[ \frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} R \right]} - \delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} \frac{\left[ \frac{A_y}{A_x} R \right]}{\left[ \frac{A_y}{A_x} L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} R \right]} \tag{16}$$

$$\frac{\delta Q_x}{Q_x} = -\delta \frac{A_y}{A_x} \frac{A_x}{A_y} - \delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x} \frac{1}{L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}} \tag{17}$$

Rearranging (17):

$$\frac{\delta Q_x}{Q_x} = -\left( \frac{\delta A_y}{A_y} - \frac{\delta A_x}{A_x} \right) - \frac{\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}}{L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}} \tag{18}$$

Since identifying assumption is Hicks-neutral, the level of technology is assumed to be constant (see, Acikgoz and Mert (2015)):

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}}{L_x^{\frac{(-b_x-1)(\beta_y+\alpha_y)}{-b_y-1}-\beta_x-\alpha_x}} \tag{19}$$

Since there are full-employment and constant returns to scale conditions at steady-state equilibrium (see, Mert (2016b)) (19) becomes:

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{(-b_x-1)}{-b_y-1}-1}}{L_x^{\frac{(-b_x-1)}{-b_y-1}-1}} \tag{20}$$

Rearranging (20):

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{b_y - b_x}{-b_y - 1}}}{L_x^{\frac{b_y - b_x}{-b_y - 1}}} \tag{21}$$

Since $\beta_x / \alpha_x = -b_x$ and $\beta_y / \alpha_y = -b_y$ (21) becomes:

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{\frac{\beta_x \alpha_y - \beta_y \alpha_x}{\alpha_y \alpha_x}}{\frac{\beta_y}{\alpha_y} - 1}}}{L_x^{\frac{\frac{\beta_x \alpha_y - \beta_y \alpha_x}{\alpha_y \alpha_x}}{\frac{\beta_y}{\alpha_y} - 1}}} \tag{22}$$

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{\frac{\beta_x \alpha_y - \beta_y \alpha_x}{\alpha_x}}{\beta_y - \alpha_y}}}{L_x^{\frac{\frac{\beta_x \alpha_y - \beta_y \alpha_x}{\alpha_x}}{\beta_y - \alpha_y}}} \tag{23}$$

Assuming $\alpha_x = \alpha_y$ as in Mert (2016a):

$$\frac{\delta Q_x}{Q_x} = -\frac{\delta L_x^{\frac{\beta_x - \beta_y}{\beta_y - \alpha_y}}}{L_x^{\frac{\beta_x - \beta_y}{\beta_y - \alpha_y}}} \tag{24}$$

(24) shows that if there is positive economic growth stem from an increase in the labor, then it should be:

i) if $\beta_x > \beta_y$ then $\alpha_y > \beta_y$ ,

ii) if $\beta_x < \beta_y$ then $\alpha_y < \beta_y$ .

Since $\alpha_x = \alpha_y$, it should be:

i) if $\beta_x > \beta_y$ then $\alpha > \beta_y$ ,

ii) if $\beta_x < \beta_y$ then $\alpha < \beta_y$ .

As a result following proposition is proved:

**Proposition 1**: For an economy with two commodities; if there are constant returns to scale conditions for the growing sector, if the identifying assumption is Hicks-neutral, and if elasticity of output with respect.. to labor is same between sectors, then, for a positive economic growth in one sector which is based on an increase in the labor of that sector, it should be:

i) if $\beta_x > \beta_y$ then $\alpha > \beta_y$ ,

ii) if $\beta_x < \beta_y$ then $\alpha < \beta_y$ .

This proposition is compatible with the **concave** production possibility frontier. According to Mert (2016), if $\beta_x > \beta_y$ and $\alpha \neq \beta_y$ and if there are i) constant returns to scale <u>only for</u> producer of $x$ $(\beta_x + \alpha = 1)$or <u>only for</u> producer of $y$ $(\beta_y + \alpha = 1)$ and if $\beta_y < \alpha$ then production possibility frontier will be concave. Moreover, if $\beta_x < \beta_y$ and $\alpha \neq \beta_y$ and if there are i) constant returns to scale <u>only for</u> producer of $x$ $(\beta_x + \alpha = 1)$or <u>only for</u> producer of $y$ $(\beta_y + \alpha = 1)$ and if $\beta_y > \alpha$ then production possibility frontier will be concave. Since, this study explains economic growth as an increase in the production capacity of *x*, we need to assume that there are constant returns to scale <u>only for</u> producer of $x$ $(\beta_x + \alpha = 1)$ .

Then Proposition 1 can be rewritten:

**Proposition 2**: For an economy with two commodities; if there are constant returns to scale conditions, if the identifying assumption is Hicks-neutral, and if elasticity of output with respect to labor is same between sectors, then, for a positive economic growth in one sector which is based on an increase in the labor of that sector, it should be:

i) if $\beta_x > \beta_y$ then $\alpha > \beta_y$ and $\alpha + \beta_y \neq 1$ ,

ii) if $\beta_x < \beta_y$ then $\alpha < \beta_y$ and $\alpha + \beta_y \neq 1$ .

## 4. Conclusion

Economic growth is frequently explained by an increase quantity of production. However, economic growth is an increase in the productive capacity of an economy and it is explained by an outward shift of **concave** production possibility frontier. This study proposes to measure economic growth as an increase in the productive capacity using production possibility frontier. Our result shows that there is a condition in order to guarantee positive economic growth such as shown in Proposition 2.

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