INTEGRATED WORLD EQUILIBRIUM WITH CONE OF COMMODITY PRICE

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Abstract – This research note studies general equilibriums by a new approach, the cone of commodity price, for the Heckscher-Ohlin model. The cone of commodity price is the counterpart concept of the cone of diversifications of factor endowments. The study presented the cone of commodity price on the IWE diagram, which displayed the trade-price relationship clearly.

Keyword: Factor Price Equalization, Factor Price Localization, Heckscher-Ohlin Model, General Equilibrium, Cone of Commodity Price, Cone of Diversification.

1. Introduction

There are many popular diagrams to illustrate the factor-price equalization (FPE) and general equilibrium for the Heckscher-Ohlin model.

McKenzie (1955) generalized the FPE to multiple factors and multiple countries. He proposed a unique geometric description for a set of all factor endowments, the cone of diversifications.

Lerner (1952) used unit-value isoquants and unit isocost lines to determine the factor price in his diagram.

Dixit and Norman (1980)’s integrated world economy (IWE) diagram provides an easy and comprehensive approach to understand trade equilibriums. It is a very important idea to study trade equilibrium and equalized factor price. The mobility of factors in their IWE box actually is an important reference for the solution of general equilibrium. By using the IWE diagram, Guo (2017a, 2017b) provided the general equilibrium for Heckscher-Ohlin model analytically. This study added the cone of commodity price on the IWE diagram to display the trade-price relation triangularly. It is very helpful to understand the general equilibrium of trade.

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² Factor prices \( w \) and \( r \) are measured by the relative price \( p^*_2 \).
³ Guo (2018) demonstrated that at this share of GNP, both countries obtained maximum redistributed shares of GNP, \( \alpha, \beta \), separately. This is the trade equilibrium point by market.
2. The review of the cone of commodity price

Under the normal assumptions of Heckscher-Ohlin theory, we denote the 2 x 2 x 2 model as the followings.

\[ AX^h = V^h \]  \hspace{1cm} (1)
\[ A'W^h = P^h \]  \hspace{1cm} (2)

where \( V^h \) is the 2x1 vector of factor endowments, its elements are capital \( K^h \) and labor \( L^h \), \( h \) indicates country, \( h = H, F \); \( X^h \) is the 2x1 vector of output; \( W^h \) is the 2x1 vector of factor prices; its elements are rental \( r \) and wage \( w \); \( P^h \) is the 2x1 vector of commodity prices; and \( A^h \) is the 2x2 matrix of factor input requirements with elements \( a_{ij}(W^h) \).

Fisher (2011) first used the goods price diversification cone, in his studies for the models of different technologies across countries, which is a duel, or a counterpart of the cone of diversification of factor endowments.
To illustrate the idea of the cone of commodity price, let us write the non-profit condition in vectors as

\[
\begin{bmatrix}
\alpha_{K1} \\
\alpha_{K2}
\end{bmatrix} r + \begin{bmatrix}
\alpha_{L1} \\
\alpha_{L2}
\end{bmatrix} w = \begin{bmatrix}
p_1 \\
p_2
\end{bmatrix}
\]

We place them in Figure 1. Multiplying each of these by factor payments, we obtain the unit capital costs \( r(a_{K2}, a_{K1}) \) and labor costs \( w(a_{L2}, a_{L1}) \). Summing these as in equation (3), we obtain the commodity price \( (p_1, p_2) \). We call the space spanned by these two vectors as “the commodity price diversification cone “, or “the cone of commodity price”, labeled by cone A in Figure 1.
The cone of diversification of factor endowments answers the question what makes sure that the commodity output obtained from equation (1) are positive. Similarly, the cone of commodity price demonstrates what confirms that factor rewards obtained from equation (2) are positive. It is clear from Figure 1 that the rewards for both factors will be positive if and only if the commodity price vector \((p_1, p_2)\) lies in between the cost requirement vectors \((\alpha_{K2}, \alpha_{K1})\) and \((\alpha_{L2}, \alpha_{L1})\).

We express the cone of commodity price in algebra as followings,

\[
\frac{\alpha_{K1}}{\alpha_{K2}} > \frac{p_1}{p_2} > \frac{\alpha_{L1}}{\alpha_{L2}} \tag{4}
\]

The definition of the share of GNP of home country in world economy is

\[
s^h = \frac{P'}{X^h} \tag{5}
\]

Corresponding the cone of commodity price (4), the share of GNP of country home will be limited by

\[
s^h_b \left(\frac{\alpha_{K1}}{\alpha_{K2}}\right) = \frac{K^H}{R^F + K^H} > s^h > s^h_a \left(\frac{\alpha_{L1}}{\alpha_{L2}}\right) = \frac{L^H}{l^F + l^H} \quad h = (H, F) \tag{6}
\]

where a and b indicate two boundaries of shares of GNP, if \(\frac{K^H}{R^F + K^H} > \frac{L^H}{l^F + l^H}\).

3. IWE Diagram with Cone of Commodity Price

Figure 2 is the IWE diagram added with a cone of commodity price. The origin of factor endowment for country home is the lower left corner \(O\), for country foreign is the right upper corner \(O'\). Suppose that the allocation of the factor endowments is at point E, where the home country is capital abundant. The cone of commodity price starts at the origin \(O''\).

The shares of GNP \(s^H_b\) and \(s^H_a\) from equation (6) are indicated at the vertical unity axis and at the horizontal unity axis, separately.

We refer to area \(EC^b NC^a\) as the trade box. \(P^H\) is the vector of factor content of trade. It should end at the line \(C^b C^a\).
Point $C^a$ identifies one boundary of trade box. If the trade happened at this point, commodity price lie at the cone ray $p^a$. The share of GNP of the home country is $s^H_a$.

Point $C^b$ identifies another boundary of trade box. If the trade happened at this point, commodity price lie at another cone ray $p^b$. The share of GNP of the home country is $s^H_b$.

Point $C$ is the trade equilibrium point supposed. The share of GNP of country home at this moment is $s^H$. The commodity price is $p^*$. It is the function of the share of GNP of the home country as

$$p^* = p^*(s^H) \quad (7)$$

The price $p^*$ meets the share of GNP $s^H$ at point D. The convex curve $C^a D C^b$ expresses the commodity price as a function of the share of GNP.

By the HOV studies, the factor contents of trade are

$$F_K^H = K^H - s^H K^w \quad (8)$$
\[ F_L^H = L^H - s^H L^w \]  
(9)

With the cone of the commodity price, Figure 2 shows a clear relationship between price and trade at equilibrium.

Figure 3 is an IWE diagram added with factor price.

Corresponding the cone of commodity price (4), the wage and rental limits are:

\[ \frac{1}{a_{K2}} > r^* > 0 \]  
(10)

\[ 0 < w^* < \frac{1}{a_{L2}} \]  
(11)

The factor price must lie the following vectors

\[ W^a = W^a \left( \frac{1}{a_{K2}}, 0 \right) \]  
(12)

\[ W^b = W^b \left( 0, \frac{1}{a_{L2}} \right) \]  
(13)

Figure 3 presents the “cone” of factor price spanned within \( W^a \) and \( W^b \).

\(^2\) Factor prices \( w \) and \( r \) are measured by the relative price \( p^*_2 = 1 \).
Points E, C, and O'' are at the same line. This is the scale where O'' allocated for the origin of factor price.

The trade box at Figure 3 is identified by the cone of factor price rather than by cone of commodity price. It illustrates the relationship between factor price and factor contents of trade directly.

At the middle point of the shares of GNP $s_H^a$ and $s_B^H$, 

$$s^H = s_a^H + \alpha = \frac{1}{2}(s_B^H + s_a^H) = \frac{1}{2} \frac{k_H^L + k^w L^H}{k^w l^w}$$  \hspace{1cm} (14)

the factor price is determined by world factor endowments\(^3\). The prices at the equilibrium solution are

$$r^* = \frac{l^w}{k^w}$$  \hspace{1cm} (15)

$$w^* = 1$$  \hspace{1cm} (16)

$$p_1^* = a_k1 \frac{l^w}{k^w} + a_{L1}$$  \hspace{1cm} (17)

$$p_2^* = a_k2 \frac{l^w}{k^w} + a_{L2}$$  \hspace{1cm} (18)

The trade pattern and trade volume will be

$$F^H_R = \frac{1}{2} \frac{k_H^L l^w - k^w L^H}{l^w}$$  \hspace{1cm} (19)

$$F^H_L = \frac{1}{2} \frac{k_H^L l^w - k^w L^H}{k^w}$$  \hspace{1cm} (20)

$$T_1^H = x_1^H - \frac{1}{2} \frac{k_H^L l^w + k^w L^H}{k^w l^w} x_1^w$$  \hspace{1cm} (21)

$$T_2^H = x_2^H - \frac{1}{2} \frac{k_H^L l^w + k^w L^H}{k^w l^w} x_2^w$$  \hspace{1cm} (22)

From price solution (15) through (18), we observe that world price (equalized factor price and common commodity price) are a function of world factor endowments. This is just the world price Dixit and Norman predicted, i.e. that when allocation of factor endowments change within the IWE box, factor price and commodity price remain same. The solution of the equilibrium

\(^3\) Guo (2018) demonstrated that at this share of GNP, both countries obtained maximum redistributed shares of GNP, $\alpha, \beta$, separately. This is the trade equilibrium point by market.
proofed the Heckscher-Ohlin theorem and the factor-price equalization theorem together analytically.

Conclusion

The cone of commodity price provides an alternative approach to understand the general trade equilibrium. It identifies the trade box (or the solution set of trade) at the IWE diagram that is helpful to illustrate the price-trade equilibrium.

The equalized factor price and the world commodity price are determined by the world factor endowments. The whole IWE box shares the same common commodity price and equalized factor price⁴.

Reference


⁴ Guo (2017a) demonstrated the world price at the equilibrium can ensure gains from trade for the countries participating free trade, by using logic autarky factor endowments determines autarky price. He states the solution of equilibrium as world (factor endowments) recourses determine world prices that ensure gains from trade for the countries participating free trade.


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